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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD COLLEGE.

ON THE THOMSON EFFECT AND THE TEMPERATURE COEFFICIENT OF THERMAL CONDUCTIVITY IN SOFT IRON BETWEEN 115° AND 204° C.

By Edwin H. Hall, L. L. Campbell, S. B. Serviss, and E. P. Churchill.[1]

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Introduction and Summary of Results.

A paper printed in these Proceedings for May, 1905, by the authors of the present paper gave the following results as obtained from the study of a certain specimen[2] of soft iron :

Composition : iron, 99.93% ; carbon, 0.059%.
Density : about 7.785 at 0° C.
Thermal conductivity, k : 0.1528 at 28.2° C.,
with a temperature coefficient 0.0003 (?) from 28° to 58°,
" " " " 0.0007 (?) " 13° " 87°,
the latter value being the more reliable.
Electric resistance, absolute : 11365 at 0° C.,
with a mean temperature coefficient 0.00519 from 0° to 100°.
"Thermo-electric height" with copper :

$$\text{approximately}\begin{cases}1028 \times 10^{-8} \text{ volt at } 26.6° \text{ C.,}\\ 980 \quad " \quad " \quad " \ 41.3° \text{ C.,}\\ 936 \quad " \quad " \quad " \ 54.5° \text{ C.,}\\ 870 \quad " \quad " \quad " \ 71.1° \text{ C.,}\end{cases}$$

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[1] Mr. Churchill was not engaged in the earlier part of the work described in this paper, but he worked with me during the greater part of July, 1906, in a supplementary investigation. — E. H. H.

[2] It is proverbial that the various properties of any given metal, as set down in physical tables, are usually obtained from different specimens. It has seemed to us worth while to study one particular kind of iron in as many of its different aspects as practicable. The study is still in progress.

Thomson effect coefficient, ν :

$$\text{approximately}\begin{cases} -757 \times 10^{-10} \text{ mean, from } 13^\circ \text{ to } 90^\circ, \\ -715 \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \ 51^\circ, \\ -793 \quad \text{“} \quad \text{“} \quad \text{“} \ 51^\circ \text{ “ } 90^\circ, \end{cases}$$

[a mean rise of -2×10^{-10} per degree].

We have now to add for the same specimen of iron :

Temperature coefficient of thermal conductivity between 115° and 204° C., referred to the value of the conductivity at 115° = — 0.00068, approximately. Electric resistance, absolute : 17260 at 100° and 26140 at 218.2°, with a mean temperature coefficient 0.00661 (on the basis of the 0° value of the resistance) between 100° and 218°.

Thomson effect coefficient, ν ($= \sigma \div T$, where σ is the fraction of a calorie absorbed per second in the Thomson effect by a current of 10 amperes flowing from a cross-section at temperature ($T - \frac{1}{2}$) absolute to a cross-section at temperature ($T + \frac{1}{2}$) absolute), from a combination of our new observations with our old ones, approximately (see pp. 614, 615).

$$-(717 + 2\,(T - 305)) \times 10^{-10},$$

from 32° C. to 182° C. (The rate of change of ν here indicated would make the tangent of the angle of inclination of the iron line on the ordinary thermo-electric diagram about 40 per cent greater at 182° C. than at 32° C.)

Battelli, who worked about twenty years ago with a specimen of iron which he does not describe, except as to its shape and size, got values from which we can deduce the approximate law

$$\nu = -(283 + 0.34\,(T - 326)) \times 10^{-10}$$

from 53° C. to 308° C.

Lecher, who also in his recent paper fails to describe the quality of the iron with which he worked, gets σ as -278×10^{-7}, that is, $\nu = -855 \times 10^{-10}$, at 52° C., and gives a formula which, when allowance is made according to the data of Bède, for the change of specific heat of iron with change of temperature, indicates a considerable increase, about 12 per cent, in the numerical value of ν between 50° and 130°, a nearly constant value from 130° to 160°, a slight numerical fall between 160° and 180°, and an increasing rate of fall with further rise of temperature.

It appears, then, that there is general agreement among those who have worked directly on the Thomson effect in iron that the line for this metal on the thermo-electric diagram should be one of increasing steepness from 50° C. to the neighborhood of 130° C.; though Lecher is in disagreement with others as to the general change of inclination of the line above this temperature.

It is noted that lead, in which the Thomson effect is undoubtedly very small at certain temperatures, has apparently been carefully examined only in the neighborhood of 50° C. (by Le Roux, by Haga, and by Battelli) and 110° C. (by Battelli), and the question is raised whether it is not desirable to examine copper, which is mechanically a much better material to work with than lead, very carefully through the widest practicable range of temperature, with a view to establishing a reference line for the thermo-electric diagram.

Incidentally, it is observed that the thermal conductivity of calcined oxide of magnesium, packed to a density of about 0.17 gram per cu. cm., is considerably less, perhaps 30 per cent less, at 218° C. than at 100° C.

General Arrangement of Apparatus.

The general method followed in our recent work was the same as that which was described in the paper already referred to; but, as many changes in detail were necessitated by working at a higher temperature, it seems desirable to show by means of diagrams the later form of the apparatus.

Figure 1 shows a horizontal section through the middle at the level of the two main iron bars, α and β, each 1 cm. in diameter, which were the subjects of the research. The pots are of cast iron, about 16.5 cm. long, in the direction of the bars, 27 cm. wide, and 25 cm. deep.

Around the main bars are two "guard-rings" (see Figure 2, a vertical cross-section of the apparatus), each consisting of twenty iron bars about 0.6 cm. in diameter, the primary object of which is to lessen lateral outflow of heat from the main bars. The outer guard-ring is a new feature of the apparatus.

Behind the brass nuts *n* on the main bars are, first, washers of brass; next, washers of leather prepared for plumbers' use, to bear a rather high temperature.[3] Mica and a plumber's cement, consisting of silicate

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[3] A trial which we now know to have been too short had persuaded us that this prepared leather would withstand the action of hot naphthalin. When, after a long course of experiments, the main apparatus here described was closely examined, it appeared that the leather washers had been reduced to dust. But the cement had apparently held, so that, so far as we know, there was no leakage at the flanges of the main bars during the experiments on the Thomson effect, though there was such leakage in experiments made later, in July, 1906. See p. 619.

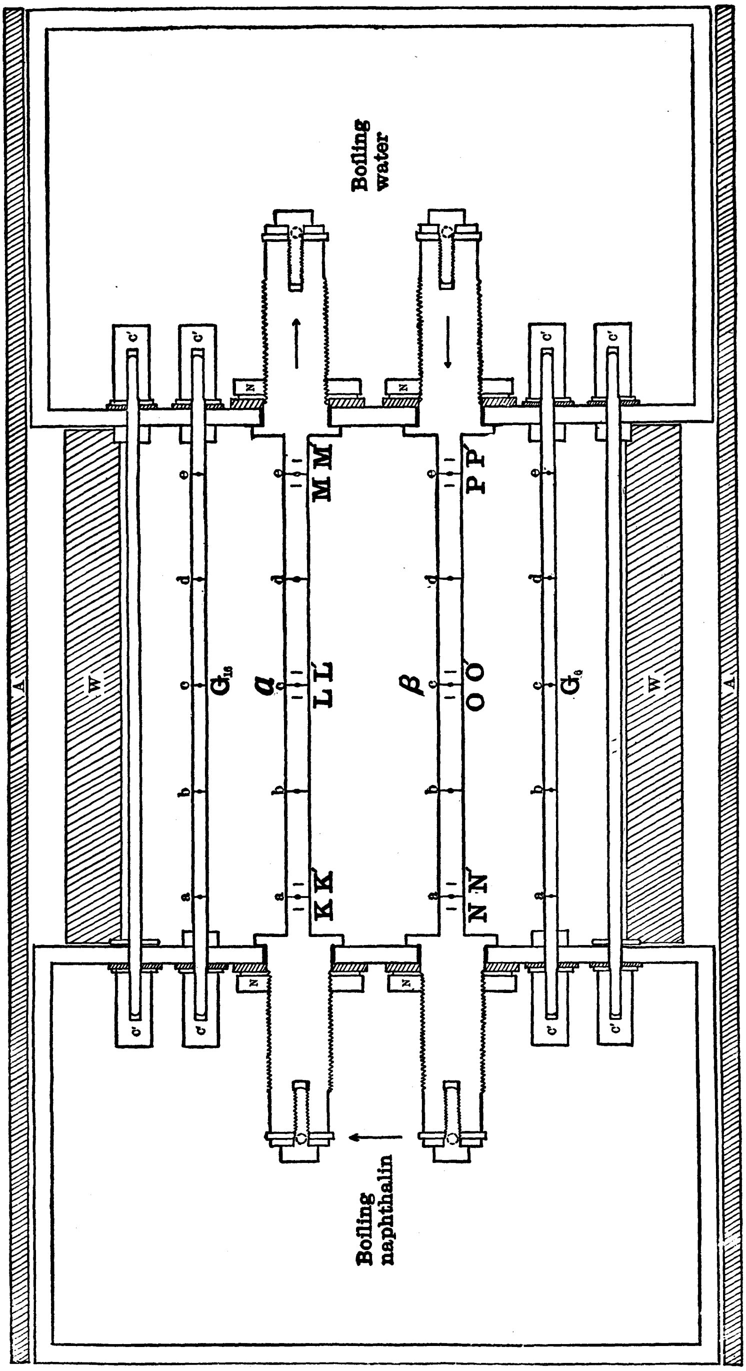
Boiling water
Boiling naphthalin
A
W
c'
N
a
b
c
d
e
G_{15}
G_6
α
β
K K'
L L'
M M'
N N'
O O'
P P'

FIGURE 1.

of soda, graphite, and asbestos, with perhaps other ingredients, were used for packing and insulation under the flanges of the main bars and within the cylindrical apertures adjacent.

To prevent leakage at the ends of the numerous guard-ring bars we depended solely on lead washers squeezed hard between the brass cap nuts *c* and the walls of the pots; and to lessen the danger of straining the apparatus and producing rupture of the joints around the main bars at the tightening of these nuts, we used on or in each guard-ring bar just outside the walls of pots, nuts or iron pins, as Figure 1 shows. Although in most cases the lead washers thus used were entirely satisfactory, we shall see later that through one or two joints leakage finally occurred.

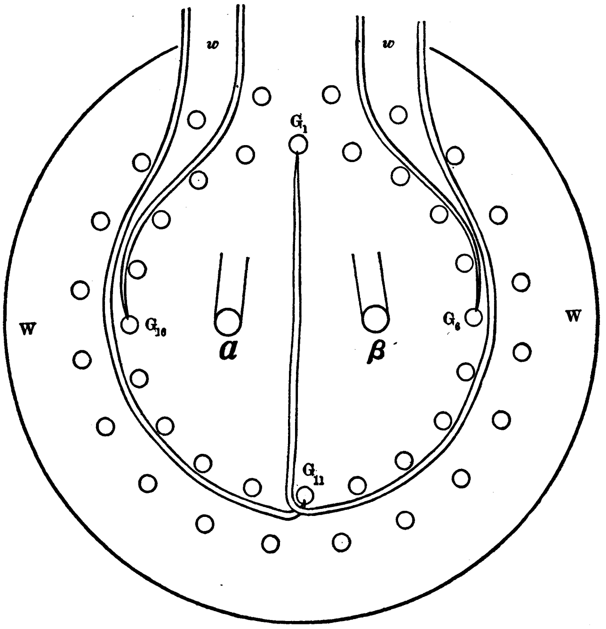

FIGURE 2.

K and K′ are the junctions, 1 cm. apart, of a copper-German-silver thermo-electric couple wound spirally in a groove made in the asphaltum coating of the bar *α*, as shown in Figure 5 of our former paper, which represents a part of the surface of the bar laid out in a plane. L L′, M M′, N N′, O O′, and P P′ are similar spirally placed couples. Each such couple is to indicate the gradient of temperature in its part of the bar on which it lies. Cemented by asphaltum to the bar α, midway between K and K′ is α_a, one junction of a copper-German-silver couple, the other junction of which is kept at a known temperature in a Crafts[4] boiler containing water or naphthalin, which apparatus is not shown in the figure. The object of this couple is to indicate the mean temperature of that part of α which lies between K and K′. The junctions α_b, α_c, α_d, α_e, β_a, β_b, β_c, β_d, and β_e have each a function similar to that of α_a; for each of them there is a corresponding junction in one or another of four Crafts boilers. The bar G_6 carries five copper-German-silver junctions, 6_a, 6_b, etc., and the bar G_{16} a like equipment, 16_a, 16_b, etc., corresponding in position and function to the junctions α_a, β_a, etc., already noted, on the main bars. In every case the junction indicated in the figure is connected with a mate, which can be kept at a definite temperature in one of the Crafts boilers. Each junction attached to a guard-ring bar is really lodged in a small hole drilled in the bar, insulation being secured by means of thin-walled glass tubing, and fixity of position by means of asphaltum varnish. Bars G_1 and G_{11}, shown in Figure 2, are similarly provided. We have, accordingly, on the main bars and the guard-ring taken together, six *a* junctions, six *b* junctions, etc., each set of six lying approximately, though by no means accurately, in one cross-sectional plane. The effect of displacements from such a plane, though for the most part eliminated in the end by the practice of reversing the gradient of temperature in the apparatus, will require notice in some cases.

In every case the distance from *a* to *b*, from *b* to *c*, etc., was intended to be 4 cm. The distance from pot to pot was very nearly 21 cm.

The copper and German-silver wires, all about 0.02 cm. in diameter, leading up from the various thermo-electric junctions which have been described, were insulated, where insulation seemed necessary, by passing them through short, thin-walled glass tubes. The wires from the main bars ran almost straight up, as Figure 2 indicates, while those from the guard-ring bars were made to follow the contour of the inner guard-ring for a greater or less distance before leaving the packing with which the guard-rings are surrounded. The wires from bar G_1, at the top, were carried down to the bottom of the inner ring, as Figure 2 shows, and then brought up on its outside. The object of these precautions was to make sure that the gradient of temperature in the wires leading from any junction should be very small in the neighborhood of that junction. All the wires, after leaving the packing surrounding the bars, ran nearly straight upward to a wooden platform about 30 cm. above the outer guard-ring. Here all the copper wires were connected with larger copper wires leading off toward the du Bois and Rubens *Panzergalvanometer* with which all the thermo-electric currents were measured, while all the German-silver wires were continued to the Crafts boilers, about 1 meter distant.

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[4] See a paper *On Thermo-electric Heterogeneity in Certain Alloys, especially German-Silver*, by the authors of the present paper, these Proceedings, March, 1906, for an account of the calibration of thermo-electric couples by use of the Crafts apparatus.

To obstruct the lateral flow of heat from the main bars outward, the outer guard-ring was covered, except at the top, where the wires came through, by a wrapping of magnesia and asbestos about 2.5 cm. thick, and the space within was stuffed rather loosely with wisps or wads of asbestos fibre. Concerning this packing something will be said later.

The space between the square pots was closed on both sides by thick asbestos board, between which and the packing outside the outer guard-ring there was at mid-level an interval about 2 cm. wide.

The space beneath the packed bars was walled in on all four sides by similar asbestos board, which was intended to prevent, to a great degree, circulation of air, and especially of hot gases from the large Bunsen-burner flames used beneath the pots, from passing up past the bars. The space above the bars, up to the wooden platform already mentioned, was boxed in in the same way, though there were, before April 13, two gaps near the top of this enclosure, through which escape of air from the interior was facilitated, it being at first considered unsafe to close in completely the space under the wooden platform.

STUDY OF THE LATERAL FLOW.

The first observations of this kind were made January 26–27, 1906. Both pots contained water boiling at atmospheric pressure, and the outer junctions of all the thermo-electric couples *a*, *b*, *c*, *d*, and *e* connected with the main bars and the guard-ring bars were placed in two of the Crafts boilers containing water boiling at atmospheric pressure.

The time at which boiling began is not recorded in this case. Observations to test the approach of the bars to a constant condition of temperature began about 6 P. M., January 26, and were continued about two hours, the indications being that a fairly stable condition had been attained by 8 P. M. The boiling was continued, however, till past midnight before any of the thermo-electric observations finally used were made.

The final operations were substantially as follows, though not made in the precise order here given : —

1. The couples 1_a, 6_a, 11_a, and 16_a, all the guard-ring couples belonging to the cross-section *a*, were joined in series, and the current sent by this combination through a known resistance was measured. This made it possible to find the difference between the boiling-point of water in the Crafts boilers and the mean temperature of the points *a* on the guard-rings, and so to find this mean temperature itself.

Similar observations were made with respect to the sections *b*, *c*, *d*, and *e* of the guard-rings.

2. The couples α_a, α_e, β_a, β_e, on the main bars, joined in series, were placed in opposition to the couples 1_a, 6_a, 11_a, and 16_a, joined in series, and the current sent by this combination through a known resistance was measured. This gave the means of finding the excess of the mean temperature of the points *a* and *e* on the main bars above the mean temperature of the points *a* on the inner guard-ring.

Similar observations gave the means of finding the excess of the mean temperature of the same points *a* and *e* on the main bars above the mean temperature of the points *e* on the inner guard-ring.

Further observations of a like character gave the excess of the mean temperature of the points *b* and *d* on the main bars above the mean temperature of the points *b* and the points *d*, respectively, on the inner guard-ring.

The excess of the mean temperature of the points *c* on the main bars above the mean temperature of the points *c* on the inner guard-ring was found in like manner.

3. The spiral couples K K′, M M′, N N′, P P′, on the main bars, were joined in series, and the current sent by this combination through a known resistance was measured. This made it possible to find the mean gradient of temperature at the points *a* and *e* on the main bars.

From (1) and (2) above we get the following summary of temperatures and temperature differences: —

	Mean *a* and *e* Temperature.	Mean *b* and *d* Temperature.	Mean *c* Temperature.
Main bars,	100.28° [5]	100.07°	100.06°
Guard-ring,	99.56°	98.55°	98.06°
Differences,	0.72°	1.52°	2.00°

Proceeding with these data, we find by a graphical method that the difference between the mean temperature of the main bars between the cross-sections a and e, a distance of 16 cm., exceeds the mean temperature of the inner guard-ring bars between the same two planes approximately 1.48°.

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[5] The barometer read about 77 cm. corrected, making the temperature of boiling water about 100.4° at the top of the pot, and perhaps 100.7° at the level of the main bars.

The mean temperature gradient, leading toward the middle, at the points a and e of the main bars was found, according to the evidence of (3) above, to be approximately 0.066° per cm. The diameter of each bar being 1 cm., and its thermal conductivity at 100° approximately 0.145, according to our observations of last year, we find as the amount of heat led into each bar per second, by conduction past a and e,

$$2 \times \frac{\pi}{4} \times 0.066 \times 0.145 = 0.0150 \text{ calorie.}$$

This is also, when a stable condition is reached, the amount of heat lost from each bar per second by lateral flow from the parts between a and e.

As the lateral outflow per second from each bar per 1 degree excess of mean temperature of the bar above the mean temperature of the inner guard-ring, all relating to the region between the cross-sections a and e, we have, then,

$$0.0150 \div 1.48 = 0.0101 \text{ calorie.}$$

On February 8 and 9 the lateral flow was measured again, all the pots in use now containing naphthalin boiling under atmospheric pressure, which was about 77.3 cm. The boiling temperature of high-grade naphthalin, such as was used in the Crafts boilers, is 218.8° at this pressure. That of the very cheap variety, which was used in the square pots at the ends of the bars, is not so well determined; but there was no need of an accurate knowledge of the temperature in these pots.

Naphthalin was boiling in all of the pots about 7 P. M. on February 8, and continued in that condition until after 6 A. M. of February 9, operations being continued through the night. The first thermo-electric observations recorded were made about 3 A. M., and the last about 6.20 A. M. The following statement will give some indication of the degree of constancy of temperature which was maintained during these observations. Under *Sensitiveness* are given the galvanometer deflections produced by a current of a certain constant strength used in determining the sensitiveness of the instrument; under *Spirals*, the deflections given by the couples K K′, M M′, N N′, P P′, joined in series; under *M against G*, the deflections given by the main bar a's and e's joined in series, in opposition to the guard-ring a's in series.

Time.	Sensitiveness.	Spirals.	*M* against *G*.
2 : 55 A.M	13.18	. . .	. . .
3 : 15	. . .	19.85	. . .
3 : 45	. . .	. . .	12.98
5 : 53	. . .	. . .	13.44
6 : 10	. . .	20.35	. . .
6 : 25	13.08	. . .	. . .

It appears that both the mean gradient at the spirals and the mean difference of temperature between the main bars and the guard-ring bars *increased* a few per cent during the course of the observations, keeping pace with each other pretty closely in this change. This is a sufficient approach to equilibrium for our purposes.

A summary of the results from the observations of February 9, which were in a general way similar to those of January 26 and 27, is given below. The *c* temperature of the main bars was found by direct thermo-electric observation; the other main bar temperatures were estimated from this one by the use of data furnished by the gradient measurements and the *M against G* measurements. The guard-ring temperatures were found by means of the subtraction here shown:

	Mean *a* and *e* Temperature.	Mean *b* and *d* Temperature.	Mean *c* Temperature.
Main bars . .	216.0°	214.7°	214.2°
M — *G*. . .	3.6°	6.5°	8.1°
Guard-ring .	212.4°	208.2°	206.1°

Plotting an M–G curve, we get, as the excess of the mean temperature of the main bars, between the cross-sections *a* and *e*, over the mean temperature of the corresponding part of the inner guard-ring, 6.32°.

Taking the thermal conductivity of iron to be 0.134 at 216°, an estimate which is in accord with our measurements of the temperature coefficient of thermal conductivity, we get, as the total flow of heat per second in each bar at the junctions *a* and *e*,

$$2 \times \frac{\pi}{4} \times 0.403 \times 0.134 = 0.848 \text{ calorie.}$$

Accordingly we get, as the lateral outflow from each bar per second, in the stable condition, per degree excess of mean temperature of the bar above the mean temperature of the inner guard-ring, all relating to the region between the cross-sections *a* and *e*,

$$0.0848 \div 6.32 = 0.0134 \text{ calorie.}$$

The difference between this estimated outflow, per degree difference of temperature in the neighborhood of 218° C., and the corresponding outflow in the neighborhood of 100° C., 0.0134 and 0.0101, respectively, is too large, apparently, to be readily attributed to mere errors of measurement. On the other hand, one may well doubt whether the coefficient of thermal conductivity of a packing of asbestos fibre really increases by 33 per cent in the rise of temperature from 100° to 218°, though we are unacquainted with any experiments upon this point except our own. We have already stated that the fibre was loosely packed, 1 gram, in, let us say, 15 cu. cm. It is easy to make the density of such packing several times as great as this, and the question has arisen whether there may not have been considerable currents of air through our loosely placed fibre and whether the increase in facility of lateral heat transmission with rise of temperature may not have been due to the part played by such air currents. In this state of doubt one naturally asks whether the coefficient of lateral outflow of heat at either temperature is consistent with the results obtained by previous experimenters on the thermal conductivity of fibrous materials.

Assuming that our lateral outflow is due to conduction proper, we can make from our data a tolerable estimate of the coefficient of thermal conductivity of the packing. Thus, examining Figure 2, we see that no heat should flow across the vertical between G_1 and G_{11}. Accordingly, α gives heat to the half-ring $G_1 G_{16} G_{11}$ only. The length of this arc, along the centres of the guard-ring bars, is approximately 7π cm., 22 cm. It is evident that α, with a given difference of temperature between it and the guard-ring, sends less heat to this arc than it would send to the same length of arc bent to a radius equal to the distance, about 4 cm., from the centre of α to the centre of the nearest guard-ring bar, G_{16}. On the other hand, it is evident that α gives out more heat to the half-ring in the actual case than it would give to the same half-ring, with the same difference of temperature, if it were placed at the centre of curvature of the ring, the radius of which is 7 cm. We can, therefore, make no very large mistake in assuming that, for a given difference of temperature, the lateral outflow of heat from α is equal to that which would occur between α and a guard-ring arc 22 cm. long, having a radius of curvature equal to $\frac{1}{2}(4+7)$ cm., α being at the centre of curvature. A point of some doubt just here is, how large a part of the circumference of α we should consider as giving off heat to the guard-ring in the imagined arrangement. This part should evidently be more than half. A considerable variation in this particular would make no great difference in the estimate of the total outflow from a to the guard-ring. We shall make no serious error if we assume that the outgiving arc of a is the same part of a circumference as the receiving arc, 22 cm. bent to a radius of 5.5 cm. As the radius of a is 0.5 cm., we take 2 cm. as the length of the effective arc on a in the case imagined. In circular measure the length of each arc is $1.27\,\pi$.

In the case of steady flow, if q represents the number of calories transmitted per second from 1 cm. length of a to the corresponding 1 cm. length of guard-ring, and if k stands for the thermal conductivity of the packing, while T stands for temperature, we have for any ring of radius r and width dr,

$$-k \times 1.27\,\pi r \frac{dT}{dr} = q, \quad \text{or} \quad dT = -\frac{q}{k \times 4} \cdot \frac{dr}{r}.$$

Integrating between limits we get

$$t_1 - t_2 = \frac{q}{4k} \log_e \frac{r_2}{r_1} = 0.6\,q \div k.$$

At 100° for $t_1 - t_2 = 1$ we have $q = 0.0101 \div 16 = 0.00063$, whence $k_{100} = 0.00038$.

At 218° for $t_1 - t_2 = 1$ we have $q = 0.0134 \div 16 = 0.00084$, whence $k_{218} = 0.00050$.

Of course no great accuracy can be claimed for a result obtained by so many rude approximations; but it seems unlikely that these approximations have introduced an error so great as 25 per cent into the estimated values of k.

If now we seek to compare these values of k for loosely packed asbestos fibre with the values obtained by other investigators, we fail to find any information given in the 1905 edition of *Landolt and Börnstein* concerning the thermal conductivity of this material. If we look at the values of k given for other fibrous materials, we see figuring prominently in tables of physical constants the values obtained many years ago by Professor George Forbes for haircloth, cotton-wool (divided), cotton-wool (pressed), flannel, and coarse linen. These values range from 0.0000402 for haircloth to 0.0000298 for coarse linen. They at first seem to make the values obtained above for asbestos packing rather improbable and tend to strengthen the suspicion of considerable air currents through this packing. But the values found by Forbes are, in all probability, decidedly too small, for they are considerably less than the fairly well-established value, about 0.00005, of the thermal conductivity of "stagnant" air at ordinary temperatures. Rubner[6] found $k = 0.000098$ for cotton-wool packed closely, about 1 gm. to 8 cu. cm., though he found smaller values for wool and silk. Lees and Chorlton[7] found $k = 0.00043$ for asbestos-paper. They give no description of this paper beyond its name and the thickness, about 0.3 cm., of the layer composed of it. The mean temperature of this layer was about 87°. The value 0.00043 lies between the two, 0.00038 and 0.00050, derived above for the effective k of our layer of asbestos fibre; but, as it is altogether likely that the asbestos-paper of Lees and Chorlton made a much more dense layer than the one which we used, and would therefore have a greater conductivity proper, it seems probable that there was more or less convective movement of air through the layer of fibre as we used it.

The coefficient of outflow through this layer at 100° was about 1.7 times as great as that found in our previous work with closely packed cotton-wool at 100°.

In making allowances for the lateral outflow at temperatures intermediate between 100° and 218° we assumed that the coefficient of lateral flow, which may be defined as *the fraction of a calorie lost per second from unit length of a main bar per degree difference of temperature between main bar and guard-ring*, increases at a uniform rate with rise of temperature from 100° to 218°.

It is assumed, furthermore, that this coefficient is practically independent of the gradient of temperature which exists in the bars and in the packing parallel to the length of the bars. This is equivalent to the assumption that the various currents of heat through the packing, whether maintained by conduction proper or by convection, are practically independent of each other. As the currents of air through the packing must be comparatively slow, this assumption is not, perhaps, a violent one, though the doubt attaching to it is sufficient, in view of the large corrections which must be made for lateral flow, to make these corrections the least satisfactory part of our work. But see p. 616.

TEMPERATURE COEFFICIENT OF THERMAL CONDUCTIVITY.

On February 7, and again during the night of March 2–3, 1906, observations were made on the temperature coefficient of thermal conductivity in the main bars. During these observations there was no electric current in these bars and the two were treated as one, in this sense, that no attempt was made to determine the temperatures or the gradients of temperature on the main bars singly, mean values only being looked for. Accordingly, in what follows t_a will indicate the mean temperature of the two main bars in the cross-section a, g_a will indicate the mean gradient of temperature of the two bars at the same cross-section, etc. Similarly t'_a will indicate the mean temperature of the guard-ring bars, 1, 6, 11, and 16, in the cross-section a, etc.

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[6] Archiv für Hygiene, **24**, 320 (1895).

[7] Phil. Mag., 5th Series, **41**, 502 (1896).

The general method followed in this phase of the investigation was to establish practically fixed conditions of temperature, by keeping the boilers in action several hours before the beginning of observations for record, and then proceed substantially as follows :

1. Data were taken from which the gradients of temperature g_a and g_e could be found.

2. Data were taken from which the difference between the temperature of the main bars and the temperature of the guard-ring bars could be found for each of the five cross-sections, a, b, c, d, e. These data were obtained, in ways already sufficiently indicated, by means of the thermo-electric couples, α_a, β_a, G_{1a}, G_{6a}, G_{11a}, G_{16a}, etc.

As the various junctions lying nominally in any given cross-section do not strictly so lie, there is opportunity for very considerable errors in the estimation of the temperature differences mentioned in (2) above. It is possible to eliminate such errors almost wholly from the final result by making two complete sets of observations of the kinds indicated in (1) and (2) above, one set with the a end of the apparatus, the east end, the hotter and the other set with the e end, the west end, the hotter. The first condition existed on February 7, and the second condition on March 2 and 3.

The following table exhibits the main data obtained from the two sets of observations. The temperatures, t, given for points on the main bars, were in some cases estimated from data obtained at other times; but the possible inaccuracy of these values cannot be a source of important error. The columns headed $(t-t')$ give the excess of temperature of points on the main bars over nominally corresponding points on the guard-ring bars.

At the foot of each gradient column is given the difference of gradient; at the foot of each t column, the range of temperature; at the foot of each t-t' column, the mean value of the column.

The values in the last three columns, which are mean values taken from the preceding columns, are used in finding the temperature coefficient of thermal conductivity. In what immediately follows the subscript h will refer to the high temperature cross-section, which was at one time a and another time e, while the subscript l will refer to the low temperature section, which was at first e and later a.

We must now take account of the fact that a part of the excess of g_h over g_l is due to the lateral escape of heat from the main bars between the h and the l cross-sections. From the data given in the last column of the table above a curve is plotted from which may be found the difference of temperature of the main bars at any section between h and l. Then, with the use of the data already given concerning lateral flows, the amount of heat escaping laterally from each one of the 16 linear centimeters of each bar between the h and l cross-sections is found. The total lateral outflow as thus found, from each bar, is 0.0369, let us say 0.037, calorie per second. Taking first a tentative value of the thermal conductivity at t_h, 203.7°, and replacing this value later by one obtained by use of the data we are now dealing with, proceeding, in other words, by a method of approximation, we find that the gradient of temperature at t_h needed to carry through the h cross-section this amount of heat is 0.346, or 0.35. This we must subtract from g_h in order to find just how much of the latter gradient is necessary to transmit the amount of heat that passes through the l cross-section. But before this subtraction is made we raise the value of g_h from 5.90+, to 5.91, in order to make due allowance for the fact of expansion in the main bars. We thus get $5.91 - 0.35 = 5.56$ as the value of g_h, which is to be compared with 5.26, the value of g_l. Accordingly, if k stands for thermal conductivity, and if f

February 7.				March 2 and 3.				Means.			
	Gradient, g.	t.	$t-t'$.		Gradient, g.	t.	$t-t'$.		Gradient, g.	t.	$t-t'$.
a	5.92°	203.2°	2.44°	e	5.89°	204.1°	5.02°	$a\ e$	5.90+°	203.7°	3.73°
b	. . .	179.4	3.44	d	. . .	179.8	5.10	$b\ d$	. . .	179.6	4.27
c	. . .	156.7	3.71	c	. . .	158.2	4.46	c	. . .	157.4	4.09
d	. . .	135.3	1.72	b	. . .	135.5	2.16	$d\ b$	. . .	135.4	1.94
e	5.30	114.4	–0.32	a	5.22	114.3	–0.39	$e\ a$	5.26	114.4	–0.35
	0.62°	88.8°	2.20°		0.67°	89.8°	3.27°		0.64+°	89.3°	2.73+°

stands for the temperature coefficient of this property between t_l and t_h, we have

$$k_h \times g_h = k_l \times g_l$$

and
$$f = \frac{k_h - k_l}{k_l} \div (t_h - t_l) = \left(\frac{k_h}{k_l} - 1\right) \div (t_h - t_l)$$
$$= \left(\frac{g_l}{g_h} - 1\right) \div (t_h - t_l) = -0.00060+.$$

As the value of this temperature coefficient is a matter of considerable interest, and as such coefficients are notoriously difficult to determine, it seems worth while to inquire how much the two values differ which are obtainable from the data of February 7 and the data of March 2 and 3, taken separately and corrected, as well as possible, for the small displacements of the various junctions from their nominal planes of cross-section.

Measurements, which were capable of no great accuracy, have indicated that the junctions on the guard-ring bars are placed, on the average, about 0.15 cm. farther toward the east end of the apparatus than the corresponding junctions on the main bars. This displacement, with the gradient of temperature ordinarily prevailing in the bars, about 5.5, would make a difference of about 0.15 × 5.5, or 0.83, degree. The values of $t - t'$, corrected accordingly, would be $2.20 + 0.83 = 3.03$ for February 7, and $3.27 - 0.83 = 2.44$ for March 2 and 3. The values of f, worked out from the two sets of data, as thus amended would be very nearly -0.00047 and -0.00073 respectively.

The value of f found in our observations of last year on the same bars of iron, between 13° and 87°, was -0.0007, the value of k at 13° being the basis. The values of f found in this paper are for the interval from 115° to 204°, approximately, and the value of k at 115° is here taken as the basis. If we use the value of k at 13° as our basis now, we get approximately -0.00056, instead of -0.00060, as the value of f between 115° and 204°. The work on thermal conductivity described in the present paper is probably entitled to more confidence than that described in our paper of last year, though the correction for lateral flow was very much greater this year than last year.

On the whole, in view of the acknowledged difficulty of making anything like an accurate determination of f, the general accord of all our results in this phase of our work is reassuring.[8]

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[8] See the Appendix to this paper for an account of further experiments on the temperature coefficient of thermal conductivity in bars α and β.

Jaeger and Disselhorst found (see Beib. zu den Ann. der Phys. for 1901, p. 20)

Measurement of the Thomson Effect.

Our general method of procedure in getting data on the Thomson effect itself, and our method of handling these data, were so fully set forth in the paper[9] already mentioned that, inasmuch as we have made no fundamental changes in these methods, we shall not describe them in detail here; but certain modifications may well be mentioned. In the previous paper occurs the following sentence: "It will be observed that such a day's work includes six complete cycles, or runs, through all the pairs of corresponding couples from M P to K N and back, three runs with the main current in condition F [cold to hot in the *front* bar, β], and three with this current in condition B [cold to hot in the *back* bar, α]." In the work described in the present paper we have usually, though not always, made more cycles during a single heating, as the following practically complete table shows:

Date.	Cycles. B	F		Date.	Cycles. B	F	
January 31	2	2	4	March 2–3	5	3 2	10
February 5–6	3 3	6	12	April 13–14	9	5 4	18
" 16–17	5 3	4 4	16	" 17–18	4 5	8	17

The main data obtained from the experiments just indicated, and from supplementary observations made at other times on the temperature differences of the main bars, are exhibited in the table on p. 608, in which

Δ_l = the difference of temp. gradient at low temp. spirals.
Δ_m = " " " " " " middle " "
Δ_h = " " " " " high " "

The + sign indicates that the steeper gradient is in the warmer bar, and *vice versa.*

t_l = the mean of the temperatures at the low temp. spirals.
t_m = " " " " " middle " "
t_h = " " " " " high " "
τ_l = " diff. " " " low " "
τ_m = " " " " " middle " "
τ_h = " " " " " high " "

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as the value of this coefficient −0.00065 in one bar of iron and −0.00014 in another bar of iron, between 18° C. and 100° C.

[9] These Proceedings, May, 1905.

EAST END HOT.

Date.	Δ_l.	Δ_m.	Δ_h.	t_l.	t_{lm}.	t_m.	t_{mh}.	t_h.	τ_l.	τ_{lm}.	τ_m.	τ_{mh}.	τ_h.	A.	Sets of Observations.
				°	°	°	°	°	°	°	°	°	°		
Jan. 31	+0.0700	+0.0083	−0.0761	114.4	135.8	158.0	179.9	203.2	0.20	0.39	0.46	0.44	0.22	25.21	4
Feb. 5, 6, 7	+0.0704	+0.0085	−0.0767	115.0	136,4	158.7	180,7	203.7	0.20	0.39	0.46	0.44	0.22	25.34	12
April 17, 18	+0.0662	+0.0073	−0.0740	114.7	136.1	158.7	180.4	203.2	0.20	0.39	0.46	0.44	0.22	25.32	17
General Mean	+0.0682	+0.0079	−0.0752	114.8	136.2	158.6	180.4	203.4	0.20	0.39	0.46	0.44	0.22	25.31	33

WEST END HOT.

Date.	Δ_l.	Δ_m.	Δ_h.	t_l.	t_{lm}.	t_m.	t_{mh}.	t_h.	τ_l.	τ_{lm}.	τ_m.	τ_{mh}.	h.	A.	Sets of Observations.
				°	°	°	°	°	°	°	°	°	°		
Feb. 16, 17	+0.0655	+0.0049	−0.0709	115.1	136.7	160.4	181.3	205.2	0.20	0.41	0.50	0.47	0.21	25.28	16
March 2, 3	+0.0609	+0.0023	−0.0681	114.7	136.3	159.9	180.8	204.6	0.20	0.41	0.50	0.47	0.21	25.26	10
April 13, 14	+0.0660	+0.0043	−0.0721	114.4	136.4	159.2	180.9	204.2	0.19	0.40	0.48	0.43	0.24	25.29	18
General Mean	+0.0647	+0.0041	−0.0708	114.7	136.5	159.8	181.0	204.6	0.196	0.41	0.49	0.45	0.22	25.28	44

t_{lm} = mean of temp's midway between the l and the m spirals.
t_{mh} = “ “ “ “ “ “ m “ h “
τ_{lm} = diff. “ “ “ “ “ l “ m “
τ_{mh} = “ “ “ “ “ “ m “ h “
A = strength of the main electric current in amperes.

In finding the "general mean" for each column except the last each quantity in the column is taken with a *weight* proportional to the number placed on its level in the last column.

The various t's and τ's in this table were not all independently observed. All those in the first part, *East End Hot*, are derived from observations made February 5 to 7, with allowance for the change which differences of barometric pressure would probably make in the temperature of the main bars, the ends of which are in boiling water or boiling naphthalin.

The t's and τ's for February 16–17 and March 2–3 are derived from observations made February 23–24. The t's and τ's for April 13–14 are derived from observations made April 14, during the same heating that gave the Δ's for these days.

In the derivation of the thermal conductivity values of iron which are needed for use with the data given above, the temperature coefficient of this conductivity is taken as —0.0007 below 115°, and as —0.0006 from this temperature up to 205°. We thus get approximately, k_l being the conductivity at t_l, etc.:

k_l	k_m	k_h
0.1436	0.1396	0.1355

The specific electric resistance of the iron, which also is needed in the evaluation of the Thomson effect, was measured by Mr. Campbell with the aid of the Crafts boilers, in the summer of 1905, and was found to be 17,260 at 100° and 26,140 at 218.2°, with a mean temperature coefficient 0.00661 (on the basis of the 0° value of the resistance) between 100° and 218°.

In the following table q' indicates the first approximation to the "Thomson effect heat"; that is, the difference between the amount of heat generated per second by the electric current between the l and the h cross-sections of either bar and the amount accounted for by Joule's law between the same cross-sections, q' being calculated by the formula (4), p. 25, of our previous paper.[10] The value q'', the second approximation to the quantity desired, obtained from q' by taking account of the difference of thermal conductivity due to the slight excess of temperature of one main bar over that of the other, differs so little from q' that it is not given in the table. Under q is given the final value of the Thomson effect heat, obtained from q'' by taking account of the amount of heat which, between the l and the h cross-sections, escapes by lateral flow from the warmer bar in excess of that which escapes in the same way from the cooler bar. This correction, which is large, usually about 23 per cent of the value of q, will be discussed at some length later. Under σ is given $q \div \frac{A}{10}(t_h - t_l)$; that is, the mean Thomson effect heat per electromagnetic unit of current per degree of the range of temperature considered. Under ν is given $\sigma \div (273 + \frac{1}{2}(t_l + h_h))$, which may be called the mean Thomson effect coefficient between t_l and t_h:

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[10] These Proceedings, May, 1905.

	Date.	$t_h - t_l$.	$q' \times 10^5$.	$q \times 10^5$.	$\sigma \times 10^7$.	$\nu \times 10^{10}$.	Wt.	General Mean $\nu \times 10^{10}$.	
East end hot	Jan. 31	88°.8	799	1028	–459	–1064	4	–1045	–1019
	Feb 5–7	88.7	801	1029	–459	–1061	12		
	Ap. 17–18	88.5	766	996	–444	–1029	17		
West end hot	Feb. 16–17	90.1	745	985	–433	–999	16	–993	
	Mar. 2–3	89.9	704	945	–417	–964	10		
	Apr. 13–14	89.8	755	986	–434	–1004	18		

The difference, about 5 per cent, between ν_E, the value of ν obtained with the east end the hotter, and ν_W, the value obtained with the west end the hotter, as shown in the table just given, has been the subject of a great deal of cogitation, and a thoroughly satisfactory explanation of it has not been found. It is unlikely that the whole difference was purely accidental in the sense that it would have disappeared in a prolonged continuation of the experiments; for the smallest ν_E is larger than the largest ν_W. It is true that t_l and t_h were determined only once with the east end the hotter, which will now be called the state E; and, moreover, the conditions during this determination were not of the best, so that it may well be doubted whether the true value of $t_h - t_l$ was any greater with state E[11] than with state W. If we assume that the two values were the same, the difference between the mean ν_E and the mean ν_W will be reduced to about 3.7 per cent; but even then the smallest ν_E will be somewhat larger than the largest ν_W.

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[11] A redetermination of this quantity for condition E after the observations of April 17–18 was prevented by a partial breaking down of the apparatus, which mishap will be described later.

There is one other important datum, namely, the mean difference of temperature between the main bars, which was measured at only one heating in condition E, namely during February 5–7. This mean value, found from the observed values at the five cross-sections *a*, *b*, *c*, *d*, *e*, by plotting a curve, was 0.389°, or 0.39°. An error of about 20 per cent in this rather small quantity would account for the whole difference between the ν_E and the ν_W as found. That an error approaching this magnitude was made here is shown to be improbable by the fact that the two determinations of the main bar mean differences of temperature made with condition W gave 0.408°, or 0.41°, and 0.391°, or 0.39°, respectively.

Other possibilities which have been considered in the endeavor to account for the difference in question are the following:

1. A partial short circuit of the main electric current in the west pot when this contained naphthalin. There is no good reason for suspecting such an action. A test made on April 18, with boiling water in the west pot and boiling naphthalin in the east pot, showed that there was no serious leakage of electricity through either liquid.

2. Difference in the sensitiveness of the thermo-electric spirals on the main bars.

A difference of two or three per cent, perhaps more, in the sensitiveness of these spirals is possible. But the observations with these spirals are taken and are combined in such ways that the result obtained from a complete set of data for condition E or condition W is dependent on the mean sensitiveness rather than on the differences of sensitiveness of these spirals. We could account for the difference between ν_E and ν_W by means of variation in these spirals, if we should assume that the two *a* spirals, at the east end of the bars, are about seven per cent more sensitive at 204° and seven per cent less sensitive at 115° than the corresponding two *e* spirals at the west end of the bars. This is very improbable.

3. Dissymmetry in the shape of the bars or in the placing of the spirals upon them.

There is none that can account for the difference which we are considering. The bars were ground to 1 cm. diameter by Brown and Sharpe. The spirals are probably not all of exactly the same length, and there may be slight inaccuracies in their emplacement; but these imperfections, though they might affect slightly the value of ν, would have little or no tendency to make a difference between ν_E and ν_W.

4. Difference in the movements of air within the loose asbestos fibre

surrounding the bars, consequent upon a reversal of the gradient of temperature in the apparatus.

Already, in connection with the discussion of loss of heat by lateral outflow, the question has been raised whether this loss is merely, or practically, a function of lateral differences of temperature. If we had to do here with a true conduction only, there would be little risk in answering this question in the affirmative; but when convection, accompanying rather general movements of the air within the packing, comes into play, as it probably does to some extent, the problem becomes more difficult. But it is not easy to find any plausible reason why this longitudinal temperature gradient should be less effective in carrying off heat from the warmer bar in condition E than in condition W. It is true, that a leakage of naphthalin vapor into the packing space while condition E prevailed might reduce the effectiveness of the convection movements by replacing air by a more sluggish medium; but in the one case in which such leakage was known to occur, the value obtained for ν_E was smaller than usual, indicating, if anything concerning lateral loss, that such loss was greater on this occasion than usual. The general matter of lateral loss will be discussed once more later in this paper. It is the most troublesome part of our experimental problem, and it may be that some inequality of packing of the asbestos fibre is at the bottom of the discrepancy which we are now discussing.

5. Change of quality from one end to the other of the main bars.

Both bars were made from pieces taken from the interior of a very soft bar some 10 cm. in diameter, and any difference of composition or physical state sufficient to make such a difference in the Thomson effect with reversal of temperature gradient is very improbable. It is, perhaps, worth noting that in our work of the year 1905 with these same bars, and with the same spirals upon them, what corresponds to our ν_E of 1906 was, apparently, about two per cent greater than what corresponds to our ν_W of 1906; that is, in both 1905 and 1906 the data obtained with the marked ends (bar α is stamped with a *1* and bar β with a *2* on one end) of the bars the hotter have indicated a slightly greater value of the Thomson effect. This is probably an accidental coincidence.

This long search for an explanation of the discrepancy in question, though it has not been decisive, has served the useful purpose of passing in review a number of possible sources of error. A difference of five per cent in the values of ν_E and ν_W is of no great consequence, if we can feel reasonably sure that the mean of the two is not more than two or three per cent different from the true value of ν.

It must be confessed that this assurance is a little difficult to hold. The close scrutiny which we have given to the matter of lateral flow has left a rather troublesome doubt as to the soundness of our working assumption that this flow is not to any important extent a function of the longitudinal gradient of temperature. Upon this point we must bring to bear whatever further evidence is available.

A satisfactory mode of investigation, if it were not so extremely laborious, would be to repeat the determination of ν with other kinds or states of packing than those here used. Something like this variation of conditions we find in a comparison of our 1905 work with that of 1906, or in the former cotton-wool, closely packed in, was used instead of the rather loosely packed asbestos of 1906, and it has already been stated that the coefficient of lateral outflow, at 100°, per degree difference of temperature between the main bars and the guard-ring bars, was about 1.7 times as great with the asbestos as with the cotton. If we have made any large error in using the assumption now on trial, we should expect a marked discordance between the results obtained from the older and the more recent work. But as the two do not cover the same temperature, one lying between 13° and 90°, while the other lies between 115° and 204°, how can we, without begging the main question which this research deals with, tell what is accordance and what is discordance?

In this difficulty we get some help from the fact that in each case we have taken note of the difference of gradient at the middle cross-section, c, of the main bars, so that it is possible to make from the data of each year's research an estimate of the rate of change of ν with change of temperature, which estimate should not be greatly affected by error in regard to the absolute magnitude of the loss by lateral flow. These estimates are:

From Data of	Range of Temperature.	Inc. of ν per Degree Rise of Temp.	Mean.
1905	32° to 71°	-2.0×10^{-10}	-1.8×10^{-10}
1906	137° to 182°	-1.6×10^{-10}	

If now, taking -1.8×10^{-10} as the rate of change of ν from the mean temperature, about 53°, of the 1905 work to the mean temperature, about 159°, of the 1906 work, we reckon from the value of ν at 53° what its value at 159° should be, we get

$$-757 \times 10^{-10} - (159-53)\ 1.8 \times 10^{-10} = -948 \times 10^{-10}.$$

The value found from the data of 1906 is, as we have seen at page 610, -1019×10^{-10}, about 7 per cent larger than the value as estimated above. As an overestimate of the loss of heat by lateral flow would tend to an increase of the numerical value of ν, the test here applied, if it can be called a test, would seem to indicate that too great rather than too small a correction had been applied for lateral flow in getting the value -1019×10^{-10} for ν. Reducing the applied correction by one third of itself would reduce ν at 159° from -1019×10^{-10} to something very near -948×10^{-10}.

Further, and perhaps stronger, evidence tending to the same effect, that too large an allowance has been made for lateral flow in the work of 1906 thus far described, is now at hand. During July, 1906, a remeasurement of the temperature coefficient of thermal conductivity of bars α and β was made[12] with calcined oxide of magnesium as a packing, through which very little, if any, convection of heat could occur. The resulting value of this coefficient was -0.00076, whereas the value found when the asbestos fibre was used as a packing was -0.00060. An overestimate of the allowance for lateral flow tends to give too small a numerical value for this coefficient. Reducing the applied correction, in the case of asbestos fibre, by one fifth of itself would raise the value found for the coefficient from -0.00060 to -0.00074, which is very close to the value found with use of oxide of magnesium as a packing.

Circumstantial evidence, then, favors the opinion that the allowance which we have made for loss by lateral flow through asbestos fibre packing should be reduced about one fourth part of itself. Perhaps the best we can do under the circumstances is to combine the various estimates of the rate of change of ν with rise of temperature as follows :

	Increase of ν per 1° Rise.	Mean.
From values of ν at 32° and 71° . . . (1905)	-2.00×10^{-10}	-2.0×10^{-10}
From values of ν at 137° and 182° . . (1906)	-1.55×10^{-10}	
From values of ν at 52° and 159° . . (1905 and 1906)	-2.45×10^{-10}	

Applying this rate of change to -757×10^{-10}, the value of ν at 52°, as found in 1905, we get, as the value of ν at 157°, -971×10^{-10}. This value is about 5 per cent less than -1019×10^{-10}, the value at 159° as found from the measurements of 1906 taken alone. This discrepancy is somewhat disappointing, and it must be admitted that our values of ν may be several per cent in error at any temperature.

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[12] By Mr. E. P. Churchill and myself. See the Appendix to this paper.— E. H. H.

On the other hand, it seems to be well established that the numerical value of ν increases at a rate not very different from that indicated above, 2.0×10^{-10} per degree, in the region between 32° and 182°. As ν is proportional to the tangent of the angle of inclination of the iron line from the horizontal line on the thermo-electric diagram, the statement just made means that in the temperature region considered the iron line should be one of increasing steepness with rise of temperature. Taking the value of ν

$$\text{at } 32^\circ \text{ as } -(757 - 20 \times 2) \times 10^{-10} = -717 \times 10^{-10},$$
$$\text{and at } 182^\circ \text{ as } -(757 + 130 \times 2) \times 10^{-10} = -1017 \times 10^{-10},$$

we find that the tangent of the angle of inclination of the iron line should be about 40 per cent greater at 182° than at 32°.

In our previous paper[13] it was pointed out that Battelli's observations on iron gave data which yield the following values for ν:

-283×10^{-10}	at	53° C.	-331×10^{-10}	at	243° C.
-319	"	" 108°	-369	"	" 308°

The mean rate of increase of ν here indicated would make the value at 182° about 18 per cent greater numerically than the value at 32°.

Battelli's general method was to surround a certain length of each of two similar bars of one metal by a narrow trough containing mercury, separated from the bar by a layer of varnish or other insulating material, establish a gradient of temperature in the bars by means of ice and boiling liquids, pass a strong electric current up the gradient in one bar and down in the other, then estimate the Thomson effect from the difference in the rate of heating of the two baths of mercury. He does not give sufficient details to assure us that he took adequate account of the tendency of the mercury, which was continually stirred, to reduce the gradient of temperature, and so the Thomson effect, in the part of the bar which it surrounded. Whether the low numerical value which Battelli found for the Thomson effect in iron, as compared with that found by us or very recently by Lecher,[14] is due to this peculiarity of his method or to his having used a very different kind of iron from that which we have used, is uncertain, but it is quite possible that his relative values of ν at different temperatures were approximately correct; and the per cent rate of change which his data give for ν with rise of temperature, though less than that which we find, is large enough to establish a strong presumption in favor of representing iron on the thermo-electric diagram by a line of constantly increasing steepness from the region of 50° to the region of 300°.

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[13] These Proceedings, **41**, 36 (1905).

[14] Annalen der Physik, 4, **19**, 859 (1906).

On the other hand, the work of Lecher, to which reference has already been made, gives at 52° the value -855×10^{-10} for ν in iron, a quantity about 13 per cent greater than our value of ν at the same temperature. As to the change of ν with rise of temperature, Lecher's data seem to indicate, when account is taken of the change of specific heat of iron with change of temperature, a considerable increase, about 12 per cent, in the numerical value of ν from 50° to 130°; a nearly constant value from 130° to 160°, a slight numerical fall between 160° and 180°, and an increasing rate of fall with further rise of temperature. Indeed, according to the curve by means of which Lecher gives his results for iron, even the quantity σ, that is, νT, begins to diminish numerically with increase of T in the neighborhood of 210° C., and goes on diminishing more and more rapidly to the upper temperature limit of the observations, 441° C., where the value of ν is scarcely more than one sixth as large as the ν at 210° C.

The method of Lecher, though very different from that of Battelli, resembled it in this particular, that both were calorimetric; that is, neither method depended on the establishment of constant conditions, but each made use of a rate of change of temperature of a quantity of metal. Battelli noted the heating effect in mercury, and he observed the change of temperature during a 30-minute run of the electric current. Lecher noted the rise of temperature in the iron itself during a 30-second run of the current. In getting the curve for iron, to which reference is made above, he seems to have ignored the change of specific heat of iron with change of temperature, though he observes that a correction might be made on account of this change, and apparently refrains from making it on the ground that "the data concerning the specific heat of iron are very uncertain." The common belief appears to be that this quantity increases rather rapidly with rise of temperature, and if account were taken of this change, the course of Lecher's curve for iron and the inferences drawn from it would be seriously affected. Lecher himself recognizes other sources of possible error in his work, and it may well be doubted whether the evidence which he presents entirely offsets the force of Battelli's data, which, as we have already seen, tend to the conclusion that not only νT but ν continues to increase with rise of temperature up to at least the neighborhood of 300° C. It must be admitted that the true course of the iron line, above 200° C., on the thermo-electric diagram is uncertain.

In the case of copper and in the case of silver Lecher does make an allowance for the change of specific heat with change of temperature. He gets with the unit current (10 amperes), in copper, —

$$\sigma = +(3.01 + 0.00662\ t)\ 10^{-6},$$

and in silver, —

$$\sigma = +(7.363 + 0.00887\ t)\ 10^{-6},$$

t being the temperature on the ordinary Centigrade scale. From these equations we get for copper, —

$$\nu = \sigma \div T = +\left(\frac{12000}{T} + 66.2\right)10^{-10},$$

and for silver, —

$$\nu = +\left(\frac{49400}{T} + 88.7\right)10^{-10}.$$

Each of these equations indicates a falling value of ν with rise of temperature. Each indicates for the ordinary thermo-electric diagram a line rising from left to right, with constantly though slowly diminishing steepness. Lecher himself would probably not claim that his observations are sufficiently accurate to establish as a fact the change of curvature of these representative lines. They do, however, make it appear probable that the lines for copper and silver should be drawn nearly straight through a great range of temperature on the thermo-electric diagram, and they suggest these metals as good subjects for further and more careful study. For the very difficulty which every one of experience in the matter finds in the measurement of the Thomson effect in any metal impresses upon us the desirability of establishing some one line on the thermo-electric diagram so accurately that others can be satisfactorily determined from it by "thermo-electric height" measurements, without further recourse to direct observations of the Thomson effect.

To be sure, the lead line, taken as horizontal, has been used in this way, and it may be asked whether it is not still sufficient for the purpose indicated. All observers, so far as we know, agree in declaring the Thomson effect in lead to be very small. Le Roux,[15] who rated this effect in iron as 31 and in copper as 2, found it *sensiblement nul* in lead. At or near 50° C., Haga[16] found the effect in "pure" lead to be negative, as in iron, but *tres faible* compared with that in platinum. Battelli,[17] stimulated by the fact that Haga had found a Thomson effect in lead, undertook to measure this effect. He found it *positive*, and gives data from which we get, —

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[15] Ann. de Chimie et de Physique, 4^{me} série, t. X, 1867, p. 201. Le Roux does not, we believe, describe the quality of the lead which he used.

[16] Ann. de l'Ecole Polytechnique de Delft, t. III, 1887, p. 51. Analysis showed the lead used by Haga to contain, "beside a trace of silver, 0.093 per cent of antimony, 0.004 per cent of iron, and 0.013 per cent of zinc." It is easy to get much purer lead now.

$$\nu \text{ at } 53.0° \text{ C.} = 4.37 \times 10^{-10},$$
$$\nu \text{ “ } 108.4° \text{ C.} = 4.29 \times \text{ “ }.$$

Below 50° C. and above 110° C. we have, apparently, no direct experimental evidence concerning the magnitude of the Thomson effect in lead, and between these two temperatures we are, finding Haga on one side and Battelli on the other, in doubt as to the sign of this effect in the purest lead. It seems probable that the lead line on the thermo-electric diagram should be very nearly horizontal below 50° and above 110°; but it can hardly be said that we have a satisfactory assurance in regard to this fundamental matter. Accordingly it seems desirable to study very carefully through a rather wide range of temperature the Thomson effect in some metal which is easily obtained in a state of great purity. Lead is such a metal, but it is mechanically difficult to work with, especially at moderately high temperatures. Copper seems to be, on the whole, a better material for the most careful direct examination, with a view to the establishment of a reference line on the thermo-electric diagram.

APPENDIX.

The doubt concerning the amount and character of the lateral escape of heat through the loose asbestos fibre packing was so great that two of us[18] undertook in July, 1906, further experiments bearing on this question. By means of a rough test with a small apparatus we satisfied ourselves that closely packed asbestos would not make a satisfactory wrapping, as the flow of heat through it appeared to be even greater than through the loosely packed asbestos which we had used. Accordingly we looked for other materials capable of bearing a temperature of 218° without injury, which might serve better. Magnesium carbonate, which is used commercially as a bad conductor of heat, was found to lose much weight, doubtless from the giving up of water of composition, at 218° and to regain weight when exposed to the air at ordinary temperature. This, therefore, was rejected. The oxide of magnesium, "light calcined magnesia," on the other hand, was found to bear the heating test well, losing at first a small amount of weight, in drying, probably, but remaining constant in weight at high temperature thereafter.

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[17] Atti della Reale Accademia dei Lincei, sérîe 4, t. 3, 1887, p. 51. Battelli does not give the analysis of the lead, but states that its density at 0° C. was 11.344.

[18] Mr. Churchill and myself. — E. H. H.

Accordingly we undertook to repack our Thomson effect apparatus, using now the dried magnesium oxide, and to repeat the experiments on lateral flow through the packing as well as those on the temperature coefficient of the thermal conductivity of iron.

As we had a strong apprehension that our boilers would now leak vapor of water or of naphthalin into the packing space, though we had done what we could to stop such leaks as we had discovered, and as we feared that any condensation of vapor in the powdery oxide would be highly objectionable, we put a sort of buffer of asbestos fibre about 0.5 cm. thick against the boiler at each end of the space to be packed, trusting that the upward movement of air through this layer would carry off vapor coming into it. Then, after putting in place the wrapping of magnesia and asbestos which lies outside the outer guard-ring (see Figure 3), we poured the magnesium oxide down into the space between the many bars, striving, by tapping the outer covering and by pressing down the oxide with strips of metal, to get the light powder so well packed that it would not by subsequent settling leave empty spaces under the bars. The density of the powder as thus placed was about 0.17 gram per cubic centimeter.

After this we made, in spite of a considerable amount of leakage of naphthalin, especially toward the last, observations which resulted as follows:

Lateral outflow from each main bar per 1° *difference of temperature between the main bars and the inner guard-ring,*

0.00791	cal.	with	both	pots	at	100°,	July	17–18
0.00894	"	"	"	"	"	"	"	27
0.00843	"							
0.00579	"	"	"	"	"	218°,	"	21–22
0.00665	"	"	"	"	"	"	"	28
0.00622	"							

Temperature coefficient of thermal conductivity in iron between 115° *and* 204°, *with reference to the value at* 115°,

$$f = -0.00076, \text{ July 19 and 24.}$$

It is to be noted that in this packing the thermal conductivity appears to be about 30 per cent greater at 218° than at 100°.

No great confidence should be placed in the numerical accuracy of this indication, but the direction of the change seems to be well established.

The cause of the apparent increase in conductivity of the packing, both at 100° and at 218°, with lapse of time is not known.

The value here found for f, combined with that found for the same coefficient when asbestos packing was used, gives as a mean

$$f = -\tfrac{1}{2}\,(0.00076 + 0.00060) = -\,0.00068.$$